

Determination of Modulus of Subgrade Reaction k_s or Westergaard Modulus and Resilient Modulus M_r using the Plate Load Test

Theory behind Modulus of Subgrade Reaction k_s or Westergaard Modulus

The assumption that the footing will behave as a rigid body is usually not applicable for large and multi-column foundations, and in such conditions, engineers prefer flexible analysis using numerous available software. These computer programs often ask for an input parameter known as the “modulus of subgrade reaction”.

Structural engineers widely use this modulus in the design of slab foundations, pavements, high-rise buildings, bridge footings, rail infrastructure, over-ground tanks, windfarm foundations, temporary platforms etc. However, a geotechnical engineer is responsible for providing the appropriate subgrade reaction modulus. The appropriate estimation of subgrade reaction modulus would result in the accurate design of structures.

The modulus of subgrade reaction, k_s , (also called coefficient of subgrade reaction or Westergaard Modulus), is defined as the pressure per unit deformation of the subgrade, and it is often expressed as $\text{kN/m}^2/\text{m}$ or kN/m^3 .

Mathematically, the modulus of subgrade reaction (k_s) is expressed as:

$$k_s = \sigma / s$$

Where σ = applied pressure and s = soil settlement

The k_s value can be determined by field tests or by correlation with other tests. There is no direct laboratory procedure for determining the k_s value.

In road and airport construction, the Westergaard modulus k , with a loading plate of 762 mm diameter is calculated as follows:

$$k_{s\ 762} = \frac{\sigma_0}{s} \left[\frac{MN}{m^3} \right]$$

σ_0 is the compressive strain that corresponds to a mean settlement of $s = 1.25$ mm for the initial loading.

In case of a plate diameter of 300 mm, k_s , is corrected with a quotient of $d = 2.22$. The ideal quotient is $d = 2.54$ (the ratio of 762 mm / 300 mm). Literature, based on experimental studies, recommends the use of a quotient of $d = 2.22$ in the place of the ideal quotient $d = 2.54$.

$$k_{s\ 300} = \frac{\sigma_0}{s \times 2.22} \left[\frac{MN}{m^3} \right]$$

The latter plate diameter may be used when the layer below the loading plate is homogeneous to a depth of $1.5 \times$ plate diameter (information without liability).

Calculating the Modulus of Subgrade Reaction k_s using the Plate Load Test AX01a

It is not possible to calculate the modulus of subgrade reaction k_s from the values E_{v1} or E_{v2} only. However, when you have all values of the test - you can use the second order polynomial described in the [DIN18134 standard](#) to extrapolate modulus of subgrade k_s .

In our [Plate Load Test AX01a](#), we have a function to approximate/extrapolate k_s from the standard E_{v1} , E_{v2} load cycles using the second order polynomial.

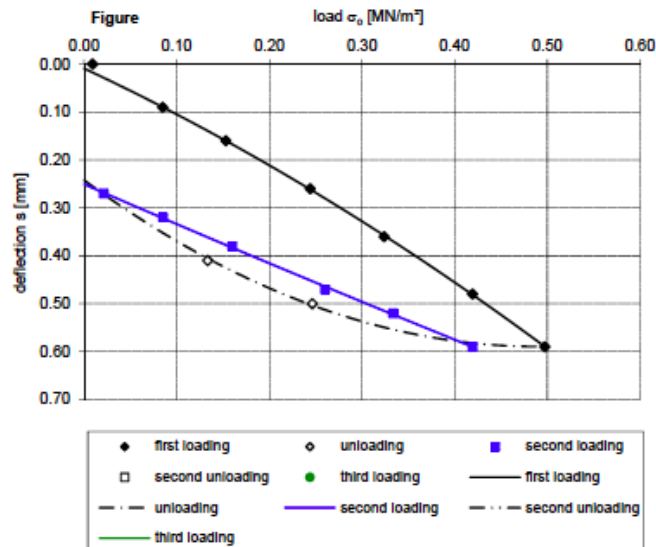
However, in case the test does not reach 1.25mm settlement, it is possible that the extrapolation failed, because sometimes there is no solution for the 2nd order polynomial at 1.25mm.

The Westergaard modulus, k_s , is determined by the software and shown in the results.

Plate Bearing Test according to DIN 18134-300

Institute:		Record number: 32
Client:	██████	Card number.: 210917101858
Project:	██████ Windfarm	Start of test: 19/3/18 9:27
Test depth: Surface		End of test: 19/3/18 9:41
Layer: FSL		Device number: 5713
Remarks:		Ø-Plate: 300 mm
Weather/Temp.: Windy 20		Lever ratio: -1:1.00
Operator: Daniel Pearce		
Equipment: Manufacturer: Anix GmbH, LVDT sensor 15mm, force sensor 100kN		

No.	Load σ_0 [MN/m ²]	Deflection s [mm]
first loading		
1	0.0092	0.00
2	0.0849	0.09
3	0.1530	0.16
4	0.2442	0.26
5	0.3239	0.36
6	0.4193	0.48
7	0.4974	0.59
unloading		
8	0.2463	0.50
9	0.1334	0.41
second loading		
10	0.0212	0.27
11	0.0851	0.32
12	0.1600	0.38
13	0.2603	0.47
14	0.3336	0.52
15	0.4193	0.59
second unloading		
third loading		



Results			
	Actual	Nominal	Rating
σ_{max} [MN/m ²]	0.4974		
E_{v1} [MN/m ²]	192.75		
E_{v2} [MN/m ²]	279.93	45	OK
E_{v2}/E_{v1} :	1.452	2.2	OK
E_{v3} [MN/m ²]			
E_{v3}/E_{v1} :			
Westergaard modulus:	328	MN/m ²	
remarks:	South East		



What is the Difference Between Resilient Modulus M_r and Elastic Modulus (Youngs Modulus)

When we load and unload the ground, a portion of the deformation may be recoverable (elastic deformation) whilst a portion may be unrecoverable (plastic deformation).

- Recoverable means the ground will return to its original shape and is in the elastic range. The load / deflection relationship is linear.
- Unrecoverable means the ground will be permanently deformed and is in the plastic range.

Modulus is a stress / strain relationship.

For Elastic Modulus (E_s) and Resilient Modulus (M_r), we are interested in the elastic range, where elastic deformation is occurring.

Elastic Modulus (sometimes referred to as Youngs Modulus)

refers to just loading and unloading a single time. With the Plate Load Test, we define this as E_{v1} for the first loading or E_{v2} for the second loading.

Resilient Modulus (M_r) refers to repeatedly loading and unloading to simulated traffic loading, for 10,000 or 100,000 cycles, for example. The resilient modulus (M_r) parameter is defined as the ratio of applied deviatoric stress to the resilient (elastic / recoverable) strain experienced by the material under repeated loadings (i.e. traffic). The Resilient Modulus is commonly used in structural design of pavements.

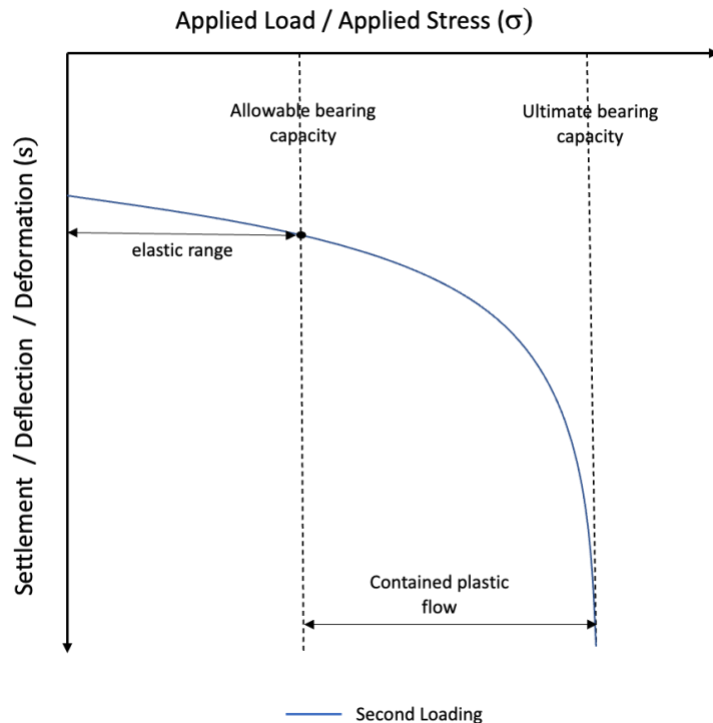
Note: **Vertical modulus** is referred to in the Austroads Guide to Pavement Design - Part 2: Pavement Structural Design. It is not defined.

Determining the Resilient Modulus M_r using the Plate Load Test

The relationship between k_s values from a Plate Bearing Test / Plate Load Test (PLT) and was updated in the fall of 2011 to better reflect published test results.

This relationship suggested in the 1993 AASHTO Guide for Design of Pavement Structures is no longer used.

$$M_r = k_s \times 19.4$$



This [web applet](#), based on the conversion factors included in [NCHRP Report 128, "Evaluation of AASHTO Interim Guide for the Design of Pavement Structures"](#), allows you to quickly estimate the subgrade resilient modulus (M_r) from either a California Bearing Ratio (CBR) or Resistance Value (R-value) measurement. Care with US units.